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Effects of directional migration on prisoner's dilemma game in a square domain

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Abstract. We introduce a new migration rule, the directional migration, into evolutionary prisoner's dilemma games defined in a square domain with periodic boundary conditions. We find that cooperation can be enhanced to a much higher level than the case in the absence of migration. Additionally, the presence of the directional migration has profound impact on the population structure: the directional migration drives individuals to form a number of dense clusters which resembles social cohesion. The evolutionary game theory incorporating the directional migration can reproduce some real characteristics of populations in human society and may shed light on the problem of social cohesion.

1 Introduction

Darwin's theory on origin of species successfully explains many natural phenomena, however, the emergence of cooperative behaviors in selfish individuals is still an open question. In the evolution of life, cooperation plays greatly active roles in different levels of complex structures of life and society. In this context, one of the most fascinating challenges is to understand how cooperation may survive in communities of selfish individuals, a problem which has been typically formalized in the framework of Evolutionary Game Theory [1]. Especially, the evolutionary prisoner's dilemma game (PDG) [2] is extensively used as a metaphor for studying cooperation between unrelated individuals. In a typical two-player, one-shot PDG, two players have to decide simultaneously cooperate or defect. A defector would gain a higher payoff when his partner cooperates, and both cooperators have the highest collective benefit shared equally between them. However, if both players decide to defect, they will be punished by a lowest payoff. In classical game theory, the result that cooperation cannot be sustained is not in line with reality. In order to figure out how cooperation emerges in nature and human society, many mechanisms supporting cooperation have been proposed during the last decades, including kin selection [3], direct reciprocity [4], indirect reciprocity [5], group selection [6], and spatial systems with short range interactions between individuals [7–16], and so on. In a spatial model, cooperation can emerge and be sustained by forming cooperative cluster, in which cooperators have more payoffs than defectors due to the support coming from their cooperative neighbors, and the most cooperation enhancement is achieved in scale-free networks because the individuals with large degrees play a key role on supporting cooperation by spreading their own strategies over their neighborhood.

Along the line of spatial evolutionary PDG, a number of other factors have been explored, including strategic complexity [17–19], adaptive network [20–26], memory effect [27], teaching ability [28–30] and social diversity [31,32]. In particular, the migration of individuals has been recognized gradually by researchers as an important factor in evolutionary PDG [33–41]. An important work considering migration of individuals was done by Vainstein et al. [34]. They first introduced random migration as a disordering factor into a square lattice and found that cooperation in the population may be enhanced by migration. Another mechanism based on migration of individuals for stabilizing and sustaining cooperation has been proposed by Helbing and Yu [35,36], which is called success-driven migration leading towards the outbreak of cooperation in a population of selfish and unrelated individuals even under noisy conditions.

Differently, Meloni et al. [37] considered a novel situation modelling highly changing environment in a number of social activities, in which individuals may move randomly on a two-dimensional plane and change their neighborhood continuously by encountering different game partners as time goes on. Considering that individuals in

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nature and human society always move to a new situation due to some reasons such as searching for food or looking for alliances purposefully rather than in a random way, we propose a new migration rule in this paper. It is named as *directional migration* where individuals are more willing to move towards the one which he just learned from. In the presence of the directional migration, we find that the cooperation level is improved strongly in the intermediate density regime compared with the case without migration. More interestingly, during the evolution, cooperators condense into a number of tight clusters similar to some kind of social cohesion which describes a well-functioning core group in which the density of individuals is high and the group members are willing to cooperate among themselves [42]. So, our study on cooperation in a population with the directional migration could shed some light on the mechanism of social cohesion.

2 Model

We consider a square domain of size $L \times L$ with periodic boundary conditions in which $N = 3 \times 10^3$ individuals can move. The area of the domain and the number of individuals are related by the density of individuals $\rho = N/L^2$. During the evolution, each individual will take three successive actions in one Monte Carlo time step. In the first action, each individual accumulates his payoff Π by performing a one-shot two-player PDG with all of his neighbors. In a PDG, the involved individuals simultaneously decide to cooperate or defect. They will receive the reward R if both cooperate and the punishment P if both defect. However, if one individual defects while the other cooperates, the defector will get the temptation T whereas the cooperator will get the sucker's payoff S. The ranking of $T > R > P > S$ and $2R > T + S$ are required by a typical PDG. Following the common practice [7], payoffs are normalized by taking $R = 1$ and $P = S = 0$. The remaining parameter $T = b$ is a controllable parameter. To determine the neighborhood of an individual, we introduce a parameter r. Two individuals are neighbors if and only if the distance between them is less than r. Accordingly, the mean connectivity, $\langle k \rangle$, in the population is related to ρ and r as $\langle k \rangle = \rho \pi r^2 - 1$. To be noted, negative $\langle k \rangle$ for sufficiently low ρ means that individuals are always isolated from others.

In the second action, individuals may update their strategies by following a certain learning rule. There are three commonly used learning rules. In the richest-following rule, the focal individual will keep his strategy unchanged if he has the highest payoff among his neighbors. Otherwise, he will learn the strategy of his neighbor with the highest payoff. In the Fermi-type rule, the focal individual randomly chooses one of his neighbors and adopts the strategy of the neighbor with a probability $W = \frac{1}{1+e^{\frac{\Delta\Pi}{K}}}$. $\Delta\Pi$ denotes the payoff difference between the focal individual and his chosen neighbor. The parameter K denotes the uncertainty in the strategy updating. The linear rule is similar to the Fermi-type rule but with a probability linearly proportional to the payoff difference. In this work, we focus on the richest-following rule. However, as we show below, the qualitative results are insensitive to the learning rules.

In the third action, individuals update their locations. Under the rule of the directional migration, the focal individual will move with a speed v towards the neighbor whose strategy he has succeeded in adopting in the second action; otherwise the focal individual does not change his location. In this action, the position of an individual i adopting the strategy of an individual j follows:

$$\overrightarrow{R}_i(t+1) = \overrightarrow{R}_i(t) + v\frac{\overrightarrow{R}_j(t) - \overrightarrow{R}_i(t)}{|\overrightarrow{R}_j(t) - \overrightarrow{R}_i(t)|}$$

with $\overrightarrow{R}_i(t)$ denoting the position of the individual i at time step t. For simplicity, we assume that the speed v is the same for all individuals. In the following, we will find that the presence of the directional migration not only favors cooperation but also gives rise to strong social cohesion.

3 Results and discussion

We first investigate how cooperation depends on the parameters such as b, ρ, r and v. To be concrete, we focus on the model with the richest-following rule as the strategy update rule. One important quantity to be monitored is the cooperator frequency F_c in the population which is defined as $F_c = N_c/N$ where N_c is the number of cooperators in the population. F_c is measured over 1000 Monte Carlo steps after 9000 transient steps and are averaged over 100 realizations.

In Figure 1, we examine F_c on the plane of b and ρ. For a comparison, we first consider the case $v = 0$, that is, the system is a motionless one. As shown in Figure 1a, for low density region (i.e. $\rho < 0.3$), most of individuals are isolated from others and F_c stays at the initial value. When ρ increases from 0.3, F_c first decreases due to the connections occasionally established among individuals which favors defectors [40]. With the further increase of ρ, the mean connectivity of the population becomes high enough to ensure the possibility of forming compact cooperator clusters through network reciprocity and, consequently, F_c starts rising. However for sufficiently high ρ, the mean field effect harmful to cooperation will be prominent since the number of neighbors of an arbitrary individual grows high enough that the approximation of the mean field becomes valid, which leads F_c to decrease with ρ. When the directional migration is introduced, the cooperation is greatly enhanced though the dependence of F_c on ρ is qualitatively unchanged. As shown in Figure 1b, the regime on the plane of b and ρ in which cooperators may survive expands greatly for the model with $v = 0.01$. For example, at $\rho = 2.5$ and $r = 1$, cooperators become extinct at $b \approx 1.4$ when $v = 0$ while the extinction of cooperators is postponed until $b \approx 1.7$ when $v = 0.01$. The enhancement of cooperation can also be observed at low ρ. For example, in the range of $\rho \in (0.6, 0.8)$ when

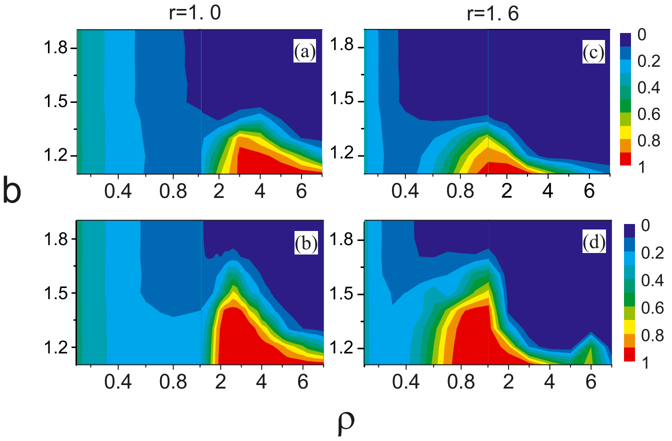

Fig. 1. (Color online) The contour plots of the frequency of cooperators F_c on the plane of b and ρ in spatial evolutionary PDGs. (a) F_c in the model without migration for $r = 1.0$; (b) F_c in the model with the directional migration for $v = 0.01$ and $r = 1.0$; (c) F_c in the model without migration for $r = 1.6$; (d) F_c in the model with the directional migration for $v = 0.01$ and $r = 1.6$.

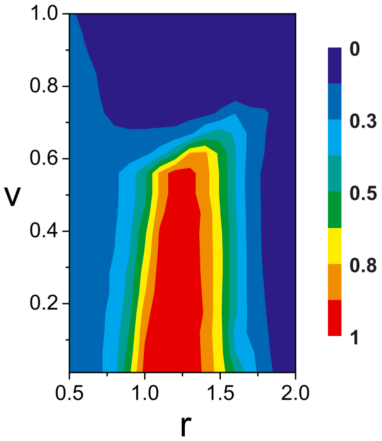

Fig. 2. (Color online) The contour plots of the frequency of cooperators F_c on the plane of v and r in evolutionary PDGs for $b = 1.3$ and $\rho = 2.0$.

$r = 1$, the improvement of cooperation level is obvious for $b < 1.4$ in comparison with the case of $v = 0$. Furthermore, the dependence of F_c on b and ρ for different r is also studied, as shown in Figures 1c and 1d. Clearly, the contour plots of F_c in Figures 1c and 1d where $r = 1.6$ look like those in Figures 1a and 1b where $r = 1$, respectively, except for a overall shift to left. The resemblance between Figures 1a and 1c or the resemblance between Figures 1b and 1d is due to the fact that the mean connectivity of the population is determined by the product of r and ρ, meanwhile F_c is strongly dependent on the mean connectivity.

Then we consider the dependence of F_c on r and v. The results are presented in Figure 2. Similar to Figure 1b, F_c displays a non-monotonic behavior against r since r is a measure on the mean connectivity of the population at a given ρ. Actually, for $\rho = 2$, the mean connectivity of the population in Figure 2 ranges from 0.6 to 23 which is almost as wide as those in Figure 1. From Figure 2, we note that the variation of F_c against r is insensitive to the velocity of the directional migration provided that v is not too high (e.g. $v < 0.6$). Such an insensitivity of F_c to v roots in the formation of dense cooperator clusters which are separated from each other, which will be elucidated in the following. However, when v is high, the fast movement of individuals induces strong mean field effect which results in the extinction of cooperators before the formation of dense cooperator clusters.

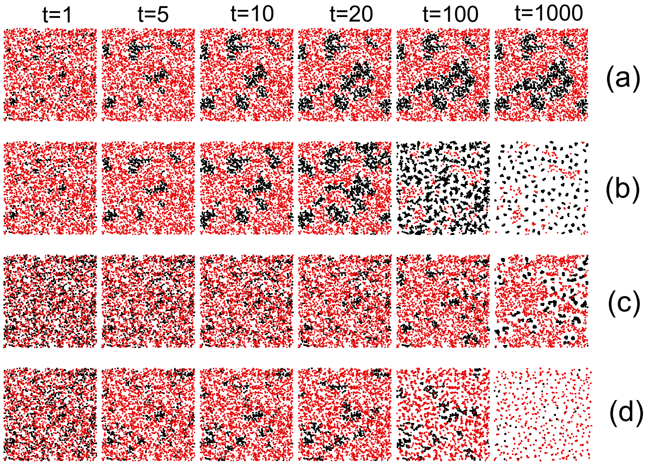

Fig. 3. (Color online) The snapshots of the strategy pattern at different time steps for $r = 1.0$. (a) The richest-following rule without migration for $b = 1.3$ and $\rho = 2.0$; (b) the richest-following rule with the directional migration for $b = 1.3$, $\rho = 2.0$ and $v = 0.01$; (c) the linear rule with the directional migration for $b = 1.1$, $\rho = 2.0$, and $v = 0.01$; (d) the Fermi-type rule with the directional migration for $b = 1.1$, $\rho = 2.0$, and $v = 0.01$. Defectors are plotted in red and cooperators in black.

The evolution of both the strategy pattern and the distribution of individuals can help us to understand clearly how cooperation evolves. For a comparison, we still consider the model with the richest-following learning rule in the absence of the directional migration (that is, $v = 0$) firstly. The successive snapshots in Figure 3a show that cooperation will rise to a certain level after a sharp fall. The maintenance of cooperation in this case is through the network reciprocity in which cooperators resist the invasion of defectors by forming cooperator clusters. Then we switch to the directional migration by setting $v = 0.01$, as shown in Figure 3b. Compared with the case $v = 0$, the snapshots in Figure 3b display one interesting feature: cooperators tend to condense into numbers of small high-connected clusters. Actually, in the early stage of the evolution, the effects of the directional migration may be ignored. As shown by Figures 3a and 3b, both situations show the similar strategy pattern for the Monte Carlo time step less than 20 where cooperator clusters have been formed. However, when time goes on, the influence of the directional migration on the distribution of individuals becomes prominent which tends to shrink the large and loose cooperator clusters into tiny and dense ones. During the process, the defectors within the interaction range of the dense cooperator clusters are turned into cooperators and dragged into the clusters, which further increases the density of those tiny clusters. In the end of the process, every individual in these tiny clusters has a connectivity around 60 which is much higher than the average connectivity around 6 in a motionless system. On the other hand, defectors outside the interaction range of the tiny dense cooperator clusters will distribute uniformly at around the initial density since no strategy update happens among them. To be mentioned, either the cooperation level or the distribution of individuals will become frozen in the end of the evolution. The similar phenomena can be found for the model with the linear rule where $b = 1.1$ (see Fig. 3c). The model with the Fermi-type rule behaves a little differently in which all individuals are inside tiny dense groups in the end. As shown in Figure 3d, each dense group is solely composed of cooperators or defectors. The presence of dense defector clusters in Figure 3d is due to the fact that individuals may learn from those with same payoff under the Fermi-type rule.

The findings in Figure 3 that the directional migration may not only enhance cooperation but also assist the formation of tiny dense clusters in a population are quite interesting. It reveals that social cohesion may be built up directly by the interplay between the imitation and the directional migration among individuals. On the other hand, the coexistence of a number of tiny dense clusters in Figure 3 actually provides a real view on the population structure in the absence of a certain central authority, which is just like the human settlement in the form of small tribes in prehistory. Additionally, even in the modern human society, we may observe that individuals tend to form small groups or organizations under the driving forces of people tending to share their neighbors' similar characters such as beliefs, interests, goals, and so on. In short, the model proposed here shed some light on the formation of social cohesion which is characterized by high cooperation level and high connections among individuals.

Now we consider the quantity, R_k, which is the ratio of the mean size of dense clusters to the average individual density. The dependence of R_k on b and ρ in Figure 4a shows that the tiny clusters are always denser than the average individual density regardless of the cooperation level. It is interesting to find in Figure 4a that the highest R_k actually occurs when cooperators become almost extinct, which indicates that the directional migration present in the transient process plays the decisive role on the formation of dense clusters. Furthermore, Figure 4a shows that there exists an optimal b in a certain range of ρ at which R_k reaches a highest value. The optimization of R_k against b can be explained by the relationship between the transient time and b. The transient process for the extinction of defectors (or cooperators) at low (or high) b is so fast that the directional migration has no time to change the distribution of individuals. Figure 4b shows the dependence of R_k on r and v. Again, provided that the directional migration is present, the tiny clusters are denser than the average individual density no matter what the cooperation level is and the highest R_k may appear in the regime where only defectors are survived.

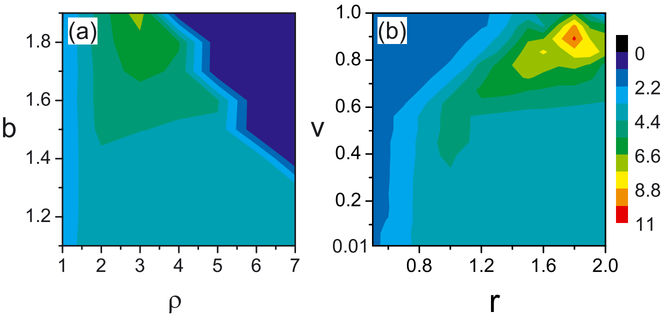

Fig. 4. (Color online) The contour plots of R_k on the plane of b and ρ for $r = 1.0$ and $v = 0.01$ in (a), and on the plane of v and r for $b = 1.3$ and $\rho = 2.0$ in (b).

Noise is ubiquitous in nature and human society. It will be valuable to investigate how noise affects the above phenomena. To do it, we introduce a random migration of individuals to the model as follows. When an individual has updated his strategy, he either performs a directional migration with $v = v_1$ with a probability p or moves at a speed of $v = v_2$ but in a random direction with a probability $1-p$. If the individual does not update his strategy, he would move at a speed of $v = v_2$ in a random direction. When $p = 0$, an extreme situation occurs where only the random migration is allowed. On the other hand, when $p = 1$, how an individual migrates depends on whether his strategy is updated. We still consider the model with the richest-following rule.

Figure 5a shows the results for the case with $p = 0$ (the pure random migration). Though the model evolves into an all cooperator state, the uniform distribution of individuals is unchanged. However, when $p \neq 0$, nonuniform distribution of individuals becomes apparent as shown by Figures 5b and 5c. In the presence of the directional migration, the random migration has two effects on the evolution of social cohesion. The first one is that the random migration tends to break a dense cluster into several small pieces, which is evidenced by Figure 6, in which the relationship between the average connectivity of cooperator clusters and the probability of directional migration p is shown. From Figure 6, we can find that more larger p, more larger the average connectivity of cooperator clusters. The second one is that the random migration may speed up the evolution of the model since the random migration of individuals outside dense clusters increases the chance that defectors are trapped into cooperator clusters. The results in Figures 5 and 6 further demonstrate that the directional migration is a necessary ingredient for social cohesion. Without directional migration, random migration cannot produce high connectivity among individuals even when cooperation in population is improved.

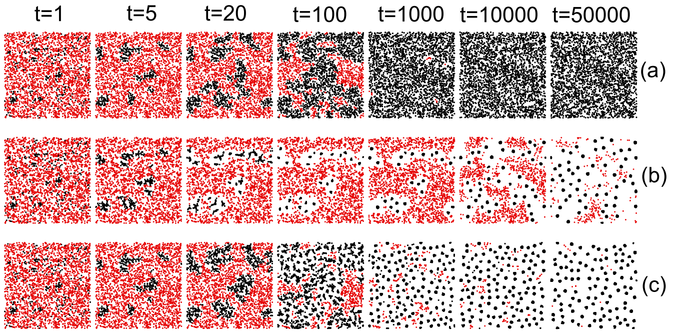

Fig. 5. (Color online) The snapshots of the strategy pattern in system with noise at different time steps for different p, the probability of individuals performing the directional migration; $b = 1.3$, $r = 1.0$, and $\rho = 2.0$; (a) $p = 0$, $v_2 = 0.01$; (b) $p = 1.0$, $v_1 = 0.1$ and $v_2 = 0.01$; (c) $p = 0.1$, $v_1 = 0.1$ and $v_2 = 0.01$. Defectors are plotted in red and cooperators in black.

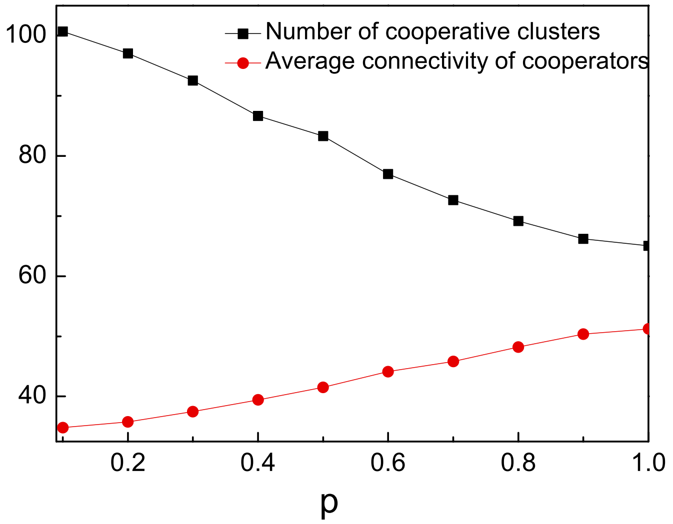

Fig. 6. (Color online) The number of cooperative clusters and the average connectivity of cooperators inside clusters are plotted against the probability of individual performing the directional migration, p. We set $b = 1.3$, $\rho = 2.0$, and $r = 1.0$.

4 Conclusion

In conclusion, we have introduced a new migration rule, the directional migration, into evolutionary PDGs in a square domain with periodic boundary conditions and studied the effects of the directional migration on cooperation in the planes of b and ρ and of v and r. In the presence of the directional migration, cooperation can be enhanced to a much higher level in comparison with the case $v = 0$. Additionally, the successive snapshots suggest that the directional migration not only enhances cooperation but also strengthens the interactions among individuals by forming a number of dense clusters which indicate the appearance of social cohesion. Whereas, the random migration introduced to individuals plays a negative role on social cohesion since the random migration always tends to break dense clusters into small pieces.

The evolution of the spatial evolutionary PDGs in the presence of the directional migration reproduces some real characteristics of populations in human society and help us to have a further understanding on the appearance of social cohesion. To be stressed, the model proposed here is only a minimal model for studying the effects of the directional migration on cooperation and social cohesion. The model can be further studied by incorporating some real factors such as to name a few, the heterogeneous interaction range in which the interaction range may be different for individuals, the collective interaction range in which the interaction range assigned to a cluster depends on the number of individuals in the cluster, and so on.

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